SOME NEW PARAMETERS ON STRONG EFFICIENT DOMINATION

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ABSTRACT

Let G = (V, E) be a simple graph with p vertices and q edges. A subset S of V(G) is called a strong (weak) efficient dominating set of G if for every v $\in$ V(G), $|N_s[v] \cap S| = 1$ ($|N_w[v] \cap S| = 1$). $N_s(v) = \{ u \in V(G) : uv \in E(G), \deg(u) \geq \deg(v) \}$.The minimum cardinality of a strong (weak) efficient dominating set of G is called strong (weak) efficient domination number of G and is denoted by $\gamma_{se}(G)$($\gamma_{we}(G)$). A graph G is strong efficient if there exists a strong efficient dominating set of G. In this paper, Subdivision number, Anti subdivision number and Subdivision stability number of a strong efficient graph are introduced. Subdivision deficiency number of a non strong efficient graph is also introduced.

Key words: Strong efficient dominating sets, Strong efficient domination number, Number of strong efficient dominating sets, Subdivision number, Anti subdivision number, Subdivision stability number and Subdivision deficiency number.

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I. INTRODUCTION

Throughout this paper, only finite, undirected and simple graphs are considered. The terms V(G) and E(G) denote respectively the vertex set and edge set of a graph G. Let G = (V, E) be a graph with p vertices and q edges. The degree of any vertex u in G is the number of edges incident with u and is denoted by deg u. The minimum and maximum degree of a graph G is denoted by δ(G) and Δ(G) respectively. A vertex of degree 0 in G is called an

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isolated vertex and a vertex of degree 1 in G is called a pendant vertex. A subset S of $V(G)$ of a graph G is called a dominating set of G if every vertex in $V(G) \setminus S$ is adjacent to a vertex in S [3]. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. Sampathkumar and Pushpalatha introduced the concepts of strong and weak domination in graphs [11]. A subset S of V (G) is called a strong dominating set of G if for every $v \in V - S$ there exists a $u \in S$ such that u and v are adjacent and deg u $\geq$ deg v. A subset S of $V(G)$ is called an efficient dominating set of G if for every $v \in V(G)$, $|N[v] \cap S| = 1$[1]. The concept of strong (weak) efficient domination in graphs was introduced by Meena, Subramanian and Swaminathan [6]. A subset S of V (G) is called a strong (weak) efficient dominating set of G if for every $v \in V(G)$, $|N_s[v] \cap S| = 1$ ($|N_w[v] \cap S| = 1$). $N_s(v) = \{ u \in V(G) : uv \in E(G), \deg(u) \geq \deg(v) \}$.The minimum cardinality of a strong (weak) efficient dominating set is called strong (weak) efficient domination number and is denoted by $\gamma_{se}(G)$($\gamma_{we}(G)$). A graph G is strong efficient if there exists a strong efficient dominating set of G. Obviously the domination number of graph changes when an edge is subdivided. This concept motivated the author to introduce the changing or otherwise of the values of strong efficient domination number. In this paper, Subdivision number, Anti subdivision number and Subdivision stability number of a strong efficient graph are introduced. The edge- magic deficiency of a graph G, denoted by $\mu(G)$, is the minimum non-negative integer n such that $G \cup nK_1$ is edge-magic [10]. This concept inspired the author to introduce the Subdivision deficiency number of a non strong efficient graph. In this paper, these new parameters are studied for some unfamiliar graphs. Terms not defined here are used in the sense of Harary [2]. For all terminologies and notations in domination, [3] is followed. The following are some basic definitions and results to discuss further.

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Definition 1.1: A graph G with vertex set $V(G) = \{v, v_{1,} v_2, \dots, v_n\}$ for $n \geq 3$ and edge set $E(G) = \{vv_i / 1 \leq i \leq n\} \cup \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_n v_1\}$is called a wheel graph of length n and is denoted by $W_{n.}$ The vertex v is called the axial or central vertex of the wheel graph.

Definition 1.2: A gear graph G_n is obtained from the wheel graph W_n by adding a vertex between every pair of adjacent vertices in the cycle.

Definition 1.3: The Bistar $D_{m,n}$ is the graph obtained from K_2 by joining m pendant edges to one end vertex of K_2 and n pendant edges to the other end of K_2. The edge K_2 is called the central edge of $D_{m,n}$ and the vertices of K_2 are called the central vertices of K_2.

Definition 1.4: A subdivision of an edge $e = uv$ of a graph G is the replacement of the edge e by a path (u, v, w). If every edge of G is subdivided exactly once, then the resulting graph is called the subdivision graph S (G).

Result 1.5 [6]: $\gamma_{se}\left(K_{1,n}\right) = 1$, n∈ N

Result 1.6 [6]: For any path P_m, $\gamma_{se}\left(P_m\right) = \begin{cases} n \text{ } if \text{ } m = 3n, n \in N \\ n+1 \text{ } if \text{ } m = 3n+1, n \in N \\ n+2 \text{ } if \text{ } m = 3n+2, n \in N \end{cases}$

Result 1.7 [6]:$\gamma_{se}\left(C_{3n}\right)$ = n, n∈ N

Result 1.8 [6]:$\gamma_{se}\left(K_n\right) = 1, n \in N$

Result 1.9 [6]:$\gamma_{se}\left(D_{r,s}\right)$ = r + 1 where r $\leq s$

Result 1.10[8]: $K_{n,n}$ −1 F is strong efficient and $\gamma_{se}\left(K_{n,n} - 1 \text{ F}\right) = 2$, ∀n∈N

Result [1.11][4]: If an efficient graph G of order n is an r-regular, then $\gamma = \frac{n}{r+1}$.

Definition 1.12[9]: Let G be a graph with a strong efficient domination number $\gamma_{se}(G)$. Number of distinct strong efficient dominating sets of a graph G is denoted by $\#\gamma_{se}(G)$.

Definition 1.13[5]: For natural numbers n and k, where n $> 2k$, a generalized Petersen graph P(n,k) has the vertex set to be the union of U = $\{u_1, u_2, \dots, u_n\}$ and V = $\{v_1, v_2, \dots, v_n\}$ and its edge set to be $\{u_i u_{i+1}, u_i v_i, v_i v_{i+k}\}$.

Result 1.14 [5]: A generalized Petersen graph P(n,k) has an efficient dominating set if and only if n ≡ $0(mod4)$ and k is odd.

Result 1.15[7]: $\gamma_{se}(G)$ =1 if and only if G has a full degree vertex.

II. MAIN RESULTS

Definition 2.1: Subdivision number of a strong efficient graph G, denoted by, $S^+(\gamma_{se}(G))$, is the minimum of the minimal number of subdivisions of G to increase the strong efficient domination number of G.

Definition 2.2: Anti subdivision number of a strong efficient graph G, denoted by, $S^-(\gamma_{se}(G))$, is the minimum of the minimal number of subdivisions of G to decrease the strong efficient domination number of G.

Definition 2.3: Subdivision stability number of a strong efficient graph G, denoted by $S^0(\gamma_{se}(G))$, is the maximum of the minimal number of subdivisions of G that will not affect the strong efficient domination number of G.

Definition 2.4: Subdivision deficiency number of a non strong efficient graph G, denoted by $D(\gamma_{se}(G))$, is the minimum of the minimal number of subdivisions of G to make it strong efficient.

Observations 2.5: The following results are easily proved.

i. $S^+(\gamma_{se}(P_m)) = 1$ if m = 3n or 3n+1, n ∈ N

= 3 if m = 3n+2, n ∈ N

ii. $S^-(\gamma_{se}(P_m)) = 1$ if m = 3n+2, n ∈ N

iii. $S^0(\gamma_{se}(P_2)) = 1$

iv. $S^0(\gamma_{se}(P_m)) = 2$ if m = 3n+1 or 3n+2, n∈ N

v. $S^+\left(\gamma_{se}(K_{1,n})\right)$ = 1, n∈ N

vi. $S^+(\gamma_{se}(C_{3n})) = 3$, n∈ N

vii. $D(\gamma_{se}(C_{3n+1})) = 2$, n∈ N

viii. $D(\gamma_{se}(C_{3n+2})) = 1$, n∈ N

ix. $S^+(\gamma_{se}(K_n)) = 2$ when n = 2

= 3 when n = 3

= 1 when n >3 and n ∈ N

x. $S^+\left(\gamma_{se}(D_{r,s})\right)$ =1 where r,s∈ N

xi. $S^-\left(\gamma_{se}(D_{r,s})\right)$ =2 where r,s≥ 2

xii. $S^0\left(\gamma_{se}(D_{r,s})\right)$ =1 where r,s≥ 2

xiii. For any wheel graph W_n, $S^+(\gamma_{se}(W_n)) = 1$, n ≥ 3

xiv. For any gear graph G_n, $S^+(\gamma_{se}(G_n)) = 3$, n ≥ 3

Definition 2.6: A crown graph on 2n vertices is an undirected graph with two sets of vertices u_i and v_i and with an edge from u_i to v_i whenever $i \neq j$.

Theorem 2.7: For any crown graph G on 2n vertices, # $\gamma_{se}(G)$ = n and $S^+(\gamma_{se}(G)) = 1$.

Proof: By Result 1.10, G is strong efficient with $\gamma_{se}(G)$ = 2. Let $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n$ be the vertices of the crown graph G. In G, $\{u_1, v_1\}.\{u_2, v_2\}, \{u_3, v_3\}$,....,$\{u_n, v_n\}$ are strong efficient dominating sets of G. Therefore # $\gamma_{se}(G)$ = n. Let u be the new vertex obtained by subdividing any edge $u_i v_j$, $1 \leq i \leq n, 1 \leq j \leq n, i \neq j$. Let the new graph obtained be G' . Since u_i dominates all the v_j's, $i \neq j$, v_i dominates all the u_j's , $i \neq j$ and also u_i and v_j are not adjacent, to form a strong efficient dominating set of G' one u_i and one v_j are needed. $u_k, k \neq i$ strongly dominates all v_j's, $v_k, k \neq j$ strongly dominates all the u_i's and u is dominated by itself. Therefore $\{u_k, v_k, u\}$ where k ≠ i, j is a strong efficient dominating set of G'. Hence G' is strong efficient and $\gamma_{se}(G') \leq 3$.Suppose S is any other strong efficient dominating set not containing u. To dominate u, either $u_i \in S$ or $v_j \in S$. If $u_i \in S$, then u_i strongly dominates all v_k's, k≠ i, j.Hence $v_i \in S$. To dominate v_j, either v_j or u_k, k≠ i, j belongs to S. If u_k, k≠ $i, j \in S$ then $|N_s[v_l] \cap S| \geq 2$, l $\neq i, j, k$ which is a contradiction. Similarly if $v_j \in S$, then $|N_S[u] \cap S| \geq 2$ which is also a contradiction. Hence no strong efficient dominating set without u exist. Therefore $\gamma_{se}(G')$ = 3 and $S^+(\gamma_{se}(G))$ = 1.

Illustration 2.8: Consider the graph G' obtained from the crown graph G on 8 vertices.

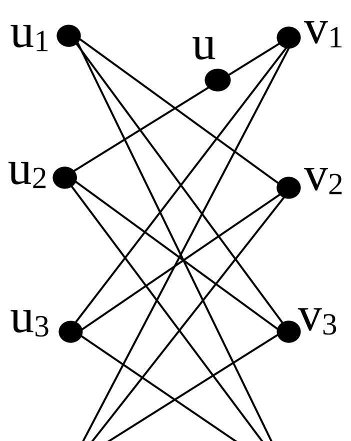

Figure 1

$\{u_3, v_3, u\}$ and $\{u_4, v_4, u\}$ are strong efficient dominating sets of G'. Hence $\gamma_{se}(G')$ = 3 and $S^+(\gamma_{se}(G)) = 1$.

Definition 2.9: A Threshold graph is a graph that can be constructed from one vertex graph by repeated applications of the following two operations.

i. Addition of a single isolated vertex to a graph
ii. Addition of a single dominating vertex to the graph.ie, a single vertex that is connected to all other vertices.

Theorem 2.10: Let G be the Threshold graph. Then $\gamma_{se}(G) = \#\gamma_{se}(G) = S^+(\gamma_{se}(G))$ = 1.

Proof: Consider a threshold graph on n vertices. Let v_n be the n^{th} vertex adjacent with the vertices $v_1, v_2, \dots, v_{n-1}$. Now v_n is the unique full degree vertex in G. By result 1.15,$\{v_n\}$ is the strong efficient dominating set of G. Therefore G is strong efficient with $\gamma_{se}(G) = 1$ and # $\gamma_{se}(G)$ = 1. Let u be the vertex obtained by subdividing the edge $v_{n-1}v_n$. In the new graph G', there is no full degree vertex in G'.Therefore $\gamma_{se}(G') \geq 2$. v_n strongly dominates all the vertices other than v_{n-1}. Also deg $(v_{n-1}) = 1$.Hence $\{v_{n-1}, v_n\}$ is a strong efficient dominating set of the new graph G'. Hence G' is also strong efficient with $\gamma_{se}(G') = 2$ and hence $S^+(\gamma_{se}(G))$ = 1.

Illustration 2.11: Consider G'obtained from the threshold graph G on 8 vertices by subdividing the edge v_7v_8.

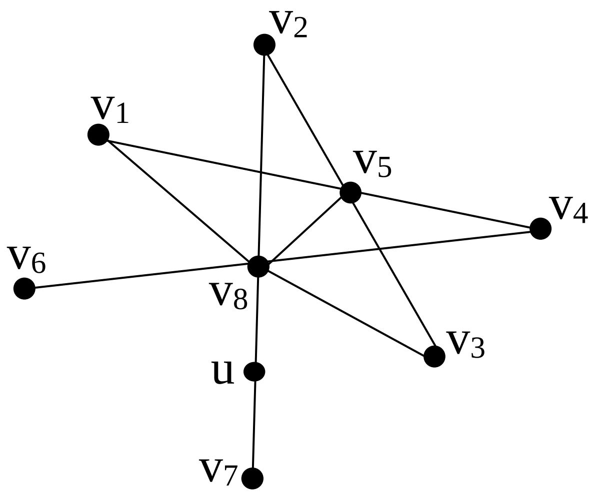

Figure 2

$\{v_8\}$ is the unique strong efficient dominating set of G. Hence $\gamma_{se}(G) = 1$ and # $\gamma_{se}(G) = 1$. $\{v_8, v_7\}$ is the unique strong efficient dominating set of G'. Hence $\#\gamma_{se}(G') = 1$and $S^+(\gamma_{se}(G)) = 1$.

Definition 2.12: A (m, n) - lollipop graph is a special type of graph consisting of an m-complete graph and an n-path connected with a bridge.

Theorem 2.13: A (m, n)- lollipop graph G is strong efficient if and only if n $\neq$ 3k where k $\in N$.

Proof: Let $v_1, v_2, \dots, v_m$ be the m vertices of K_m and $u_1, u_2, \dots, u_n$ be the n vertices of P_n. Let v_iu_1 be the bridge of the (m, n)- lollipop graph G. Suppose n$\neq 3k$.

Case(i): Let n = 3k+1. In G, deg$(v_i) =$ m $= \Delta(G)$.v_i strongly dominates $v_1, v_2, \dots, v_{i-1}, v_{i+1}, \dots, v_m$ and u_1. Vertices u_{3j}, $1 \leq j \leq k$ strongly dominates all the remaining vertices of the path P_n. Let S = $\{v_i, u_3, u_6, \dots, u_{3k}\}$. Then S is the unique strong efficient dominating set of G. Hence G is strong efficient. Therefore $\gamma_{se}(G) = 1 + k$ and $\#\gamma_{se}(G) = 1$.

Case(ii): Let n = 3k+2. Proceeding as in case (i), $\{v_i, u_3, u_6, \dots, u_{3k}, u_{3k+2}\}$ is the unique strong efficient dominating set of G. Hence G is strong efficient. Therefore $\gamma_{se}(G) = 2 + k$ and $\#\gamma_{se}(G) = 1$.

Conversely let n = 3k. Suppose G is strong efficient. Let S be a strong efficient dominating set of G. Since v_i is the unique maximum degree vertex, $v_i \in S$. v_i strongly dominates all the vertices of K_m and u_1. Hence $u_3, u_6, \dots, u_{3k-3}$ belong to S. u_{3k-3} strongly dominates u_{3k-4} and u_{3k-2}. Since deg$u_{3k} <$ degu_{3k-1}, $u_{3k-1} \in S$. But $|N_s[u_{3k-2}] \cap S| = |\{u_{3k-3}, u_{3k-1}\}| > 1$ which is a contradiction. Therefore $u_{3k-1} \notin S$. Hence there is no element in S to dominate u_{3k-1} which is a contradiction. Hence G is not strong efficient when n = 3k.

Observation 2.14: For a (m, n)-lollipop graph G,

i. $D\left(\gamma_{se}(G)\right) = 1$ when n = 3k, k ≥ 1

ii. $S^+\left(\gamma_{se}(G)\right) = 1$ when n = 3k+1, k ≥ 1

= 3 when n = 3k+2, k ≥ 1

iii. $S^0\left(\gamma_{se}(G)\right) = 2$ when n = 3k+2, k ≥ 1

Theorem 2.15: For the Petersen graph G with n = 5 and k = 1, D$\left(\gamma_{se}(G)\right) = 2$.

Proof: By result 1.14, Petersen graph G is not strong efficient. Let $v_1, v_2, \dots, v_{10}$ be the vertices of G. Consider the cycle $v_1 v_2 v_3 v_4 v_5 v_1$. Let $G^{'}$ be the graph obtained by subdividing an edge of G.

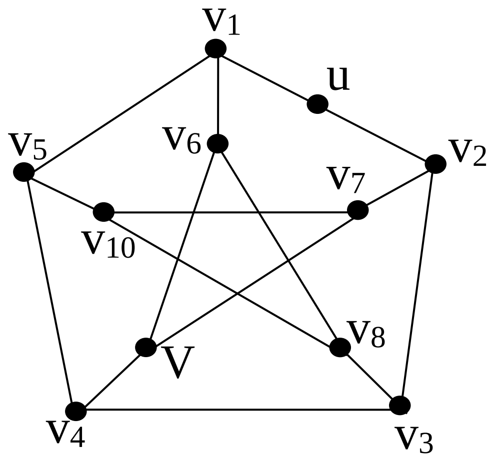

Figure 3

Case (1): Let u be the vertex subdividing any one of the edges $v_1 v_2$, $v_2 v_3$, $v_3 v_4, v_4 v_5$ or $v_5 v_1$. Without loss of generality, let u be the vertex subdividing the edge $v_1 v_2$ of G. Suppose $G^{'}$ is strong efficient. Let S be a strong efficient dominating set of $G^{'}$.

Sub case$(1a)$: Suppose v_1 or v_2 $\in$ S. If $v_1 \in$ S, then $v_1$strongly dominates v_5,$v_6$and u. If $v_3 \in$ S, then v_3 strongly dominatesv_2, v_4 and v_8. If any one of the remaining three vertices v_7 or v_9 or v_{10} $\in$ S, then $|N_s[v_2] \cap S| = |\{v_3, v_7\}| > 1$ or $|N_s[v_4] \cap S| = |\{v_3, v_9\}| > 1$ or $|N_s[v_5] \cap S| = |\{v_1, v_{10}\}| > 1$ respectively. This is a contradiction. Proof is similar if $v_2 \in$ S. Therefore $G^{'}$ is not strong efficient.

Sub case $(1b)$: Suppose v_3 or v_4 or v_5 $\in$S. If $v_5 \in$ S, then v_5 strongly dominates v_1, v_4 and v_{10}. If $v_2 \in$ S, then v_2 strongly dominates u, v_3 and v_7. If any one of the remaining three vertices v_6, v_8 and v_9 $\in$ S, then $|N_s[v_1] \cap S| = |\{v_5, v_6\}| > 1$ or

$|N_s[v_3] \cap S| = |\{v_2, v_8\}| > 1$ or $|N_s[v_4] \cap S| = |\{v_5, v_9\}| > 1$respectively. This is a contradiction. Proof is similar if v_3 ∈ S. Therefore G' is not strong efficient. On the other hand if v_4 ∈ S, then v_4 strongly dominates v_3, v_5 and v_9. To dominate v_1 andv_2, either v_1 and$v_2$belong to S or v_6 and v_7 belong to S. If $v_1 \in S$, then v_5 is strongly dominated by v_1 and v_4. If $v_2 \in S$, then v_3 is strongly dominated by v_2 and v_4. Hence v_1 andv_2 do not belong to S. If v_6 and v_7 belong to S, then v_9 is strongly dominated by $v_{4,}$ $v_6$and v_7. Hence there is no element in S to dominate v_1 and v_2. This is a contradiction. Therefore G' is not strong efficient.

Case (2): Let u be the vertex subdividing any one of the edges $v_1v_6, v_2v_{7,}v_3v_8, v_4v_9$ or v_5v_{10}. Without loss of generality, let u be the vertex subdividing the edge v_1v_6.

Suppose $v_1 \in S$. Then v_1 strongly dominates $v_2, v_5$and u. v_3 and v_4 do not belong to S, otherwise v_2 and $v_5$are strongly dominated by two verticesv_1, v_3 and v_1, v_4 respectively. If v_8 and $v_9 \in S$, to dominate v_3 and v_4 respectively, then v_6 is strongly dominated by v_8 and v_9.Hence there is no strong efficient dominating set to dominate v_3 and v_4. Proof is similar if $v_i, 2 \leq i \leq 10$ belong to S. Hence G' is not strong efficient.

Case (3): Consider the cycle $v_6uv_8v_{10}v_7v_9v_6$.

Proceeding as in case (i), G' is not strong efficient. Therefore there exists no strong efficient dominating set with $D(\gamma_{se}(G)) = 1$. Hence $D(\gamma_{se}(G)) \geq 2$.

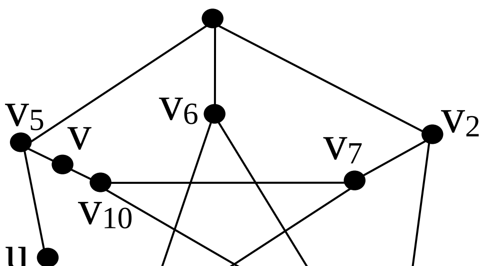

Figure 4

Let G'' be the new graph obtained by subdividing the edges $v_{i-1}v_i$ and $v_i v_{i+5}$, $2 \leq i \leq 5$ of G. Without loss of generality, let u and v be the new vertices obtained by subdividing the edges $v_4 v_5$ and $v_5 v_{10}$ respectively. Then S = $\{v_1, v_4, v_{10}\}$ is the unique strong efficient dominating set of the new graph G''. Therefore G'' is strong efficient with $\gamma_{se}(G'')$ = 3 and hence $D(\gamma_{se}(G)) = 2$.

Theorem 2.16: The 5-Regular Icosahedral graph G is strong efficient with $\gamma_{se}(G)$ = 2 and $S^+(\gamma_{se}(G)) = 1$.

Proof: Let $v_1, v_2, \dots, v_{12}$ be the vertices of the 5-Regular Icosahedral graph G.

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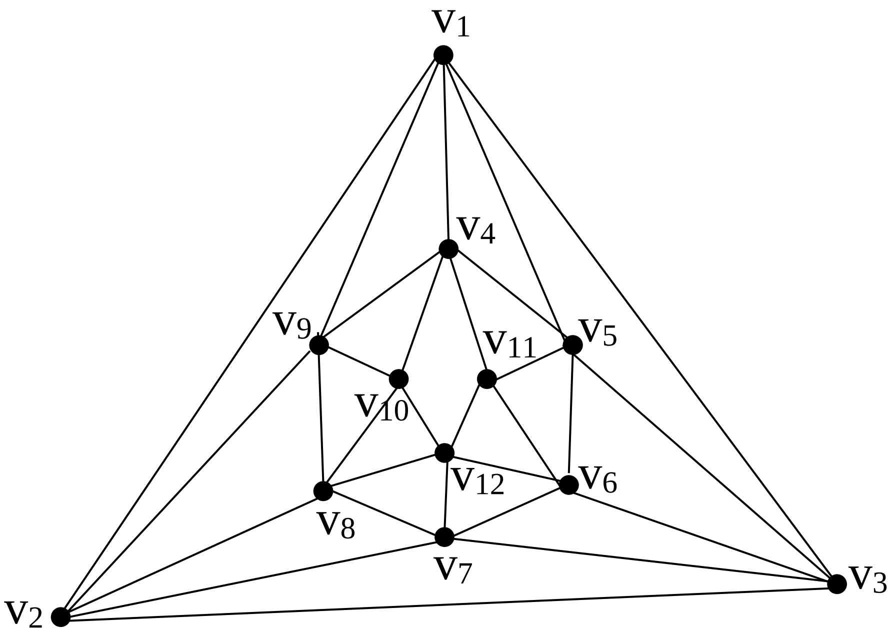

Figure 5

v_4 strongly dominates the vertices v_1, v_5, v_9, v_{10} and v_{11}. Similarly v_7 strongly dominates the vertices v_2, v_3, v_6, v_8 and v_{12}. Then S = $\{v_4, v_7\}$ is a strong efficient dominating set of G. Similarly $\{v_1, v_{12}\}, \{v_6, v_9\}, \{v_5, v_8\}, \{v_2, v_{11}\}, \{v_3, v_{10}\}$are also strong efficient dominating sets of G. Since G is regular, it is efficient and all efficient dominating sets have same cardinality. Hence $\gamma_{se}(G)$ = 2 and #$\gamma_{se}(G)$ = 6. Let G' be the graph obtained by subdividing any one of the edges v_1v_2 or v_2v_3 or v_3v_1.

Without loss of generality, let u be the vertex obtained by subdividing the edge v_1v_2. In the new graph G', as discussed earlier, v_4 and v_7 strongly dominates all the vertices other than u. Also u is the only vertex such that $\deg(u) = \delta(G') = 2$. Therefore $\{v_4, v_7, u\}$ is a strong efficient dominating set of $G'$with cardinality 3. Similarly $\{v_6, v_9, u\}, \{v_5, v_8, u\}$and $\{v_3, v_{10}, u\}$ are also strong efficient dominating sets of G'.

Claim: There exists no strong efficient dominating set without u. Suppose S is a strong efficient dominating set without u. For, if $v_1 \in S$, then it strongly dominates u, v_3, v_5, v_4 and v_9. To strongly dominate v_2, either v_7 or v_8 $\in$. If $v_7 \in S$, then v_3 is strongly dominated by v_7 and v_1. If $v_8 \in S$, then v_9 is strongly dominated by v_8 and v_1. Hence there is no element in S to dominate v_2. This is a contradiction. Proof is similar if $v_2 \in S$. Hence $\gamma_{se}(G') = 3$, $\#\gamma_{se}(G') = 4$ and $S^+(\gamma_{se}(G)) = 1$.

Note 2.17: The coxeter graph is a 3-regular graph with 28 vertices and 42 edges.

Theorem 2.18: For the coxeter graph G, $\gamma_{se}(G) = 7$ and $S^+(\gamma_{se}(G)) = 1$

Proof: Let $v_1, v_2, \ldots, v_{28}$ be the vertices of the coxeter graph G.

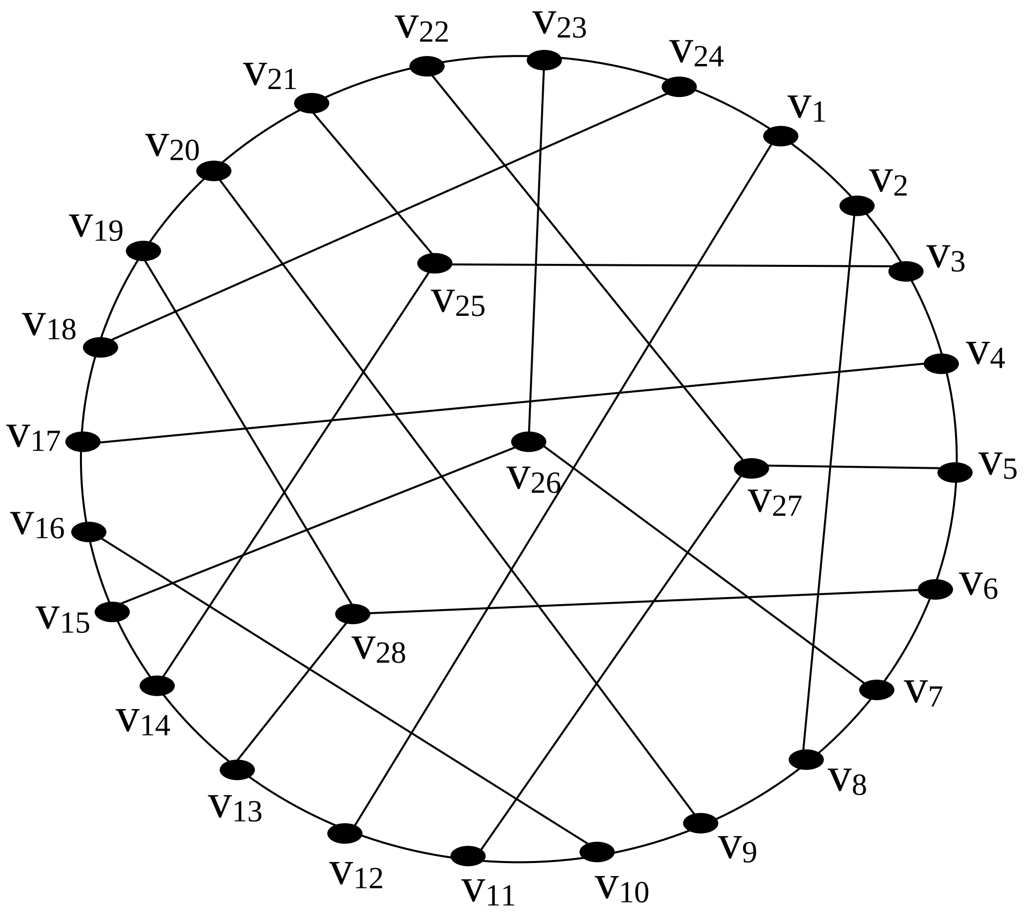

Figure 6

Let S = $\{v_1, v_9, v_{17}, v_{25}, v_{26}, v_{27}, v_{28}\}$. Then S is the unique strong efficient dominating set of G. Hence G is strong efficient and by result 1.11, $\gamma_{se}(G) = 7$. Let the vertex u subdivide any edge $v_i v_j$ such that v_i and v_j are not the elements of the strong efficient dominating set S of G. Let the new graph be G' .Without loss of generality, let the edge $v_2 v_3$ of G be subdivided by the vertex u. Also u is the only vertex in G' such that $\deg(u) = \delta(G') = 2$. Clearly S∪ $\{u\}$ is a strong efficient dominating set of G'. Hence G' is strong efficient. Suppose T is any other strong efficient dominating set of G' without u. Then either $v_2 \in T$ or $v_3 \in T$.

Case(i): Suppose $v_2 \in T$. Then v_2 strongly dominates v_1, u and v_8.

To strongly dominate v_{24} either v_{23} or $v_{18} \in T$.

Subcase(ia): Suppose $v_{23} \in T$. Then v_{23}strongly dominates v_{22}, v_{24} and v_{26}. To strongly dominate v_{21} either v_{20} or $v_{25} \in T$.

Subsubcase(iai): Suppose $v_{20} \in T$. Then v_{20} strongly dominates v_{21}, v_{19} and v_9. To strongly dominate v_{18}, $v_{17} \in T$. Now v_{17} strongly dominates v_{16}, v_{18} and v_4. To strongly dominate v_3, either v_3 or $v_{25} \in T$. In such cases, $|N_s[u] \cap T| = |\{v_2, v_3\}| > 1$ or $|N_s[v_{21}] \cap S| = |\{v_{20}, v_{25}\}| > 1$ which is a contradiction to G' is strong efficient. Therefore $v_{20} \notin T$.

Subsubcase$(iaii)$: Suppose $v_{25} \in T$. Then v_{25} strongly dominates v_3, v_{21} and v_{14}. Now v_{19} strongly dominates v_{18}, v_{20} and v_{28}. v_{16} strongly dominates v_{17}, v_{15} and v_{10}. To strongly dominate v_{13}, $v_{12} \in T$. In such case $|N_s[v_1] \cap T| = |\{v_2, v_{12}\}| > 1$ which is a contradiction to G' is strong efficient. Hence $v_{25} \notin T$. From all the above cases, $v_{23} \notin T$.

Subcase(ib): Suppose $v_{18} \in T$.Then v_{18} strongly dominates v_{24}, v_{19} and v_{17}.To strongly dominate v_{23} either v_{22} or $v_{26} \in T$.

Subsubcase(ibi): Suppose $v_{22} \in T$.Then v_{22} strongly dominates v_{23}, v_{21} and v_{27}. To strongly dominate v_{20}, $v_9 \in T$. In such case, $|N_s[v_8] \cap S| = |\{v_2, v_9\}| > 1$ which is a contradiction to G' is strong efficient. Hence $v_{22} \notin T$.

Subsubcase$(ibii)$: Suppose $v_{26} \in T$.Then v_{26} strongly dominates v_{23}, v_7 and v_{15}. v_{21} strongly dominates v_{22}, v_{20} and v_{25}. v_{18}strongly dominates v_{19}, v_{17} and v_{24}. To strongly dominate v_{16}, $v_{10} \in T$. v_{10} strongly dominates v_9, v_{11} and v_{16}.v_{13} strongly dominates v_{12}, v_{14} and v_{28}.v_5 strongly dominates v_4, v_6 and v_{27}. If $v_3 \in T$,then $|N_s[u] \cap T| = |\{v_1, v_2\}| > 1$ which is a contradiction to G' is strong efficient. Hence $v_{26} \notin T$. From all the above cases, $v_{18} \notin T$.

Case(ic): Suppose $v_4 \in T$. Then v_4 strongly dominates v_3, v_5 and v_{17}. To strongly dominate v_6 either v_7 or $v_{28} \in T$.

Subcase(ici) If $v_7 \in T$, then $|N_s[v_8] \cap T| = |\{v_2, v_7\}| > 1$ which is a contradiction to G' is strong efficient. Hence $v_7 \notin T$.

Subcase$(icii)$ Suppose $v_{28} \in T$. Then v_{28} strongly dominates v_6, v_{13} and v_{19}. v_{26} strongly dominates v_7, v_{15} and v_{23}. v_{10} strongly dominates v_9, v_{11} and v_{16}. Hence $v_{12} \in T$. Thus

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$|N_s[v_1] \cap T| = |\{v_2, v_{12}\}| > 1$which is a contradiction to G' is strong efficient. Hence $v_4 \notin T$. From all the above cases, $v_2 \notin T$.

Case(ii): Suppose $v_3 \in T$. Then v_3 strongly dominates v_{25}, u and v_4.$v_1 \in T$. $v_1$strongly dominates v_{24}, v_2 and v_{12}. Now $v_{22} \in T$. v_{22} strongly dominates v_{23}, v_{21} and v_{27}.

To strongly dominate v_{20} either v_{19} or $v_9 \in T$.

Subcase(iia): Suppose $v_{19} \in T$. Then v_{19} strongly dominates v_{20}, v_{18} and v_{28}. v_{16} strongly dominates v_{17}, v_{15} and v_{10}. Now either v_{14} or $v_{13} \in T$. In such cases, $|N_s[v_{25}] \cap T| = |\{v_3, v_{14}\}| > 1$ or $|N_s[v_{12}] \cap T| = |\{v_1, v_{13}\}| > 1$ which is a contradiction to G' is strong efficient. Hence $v_{19} \notin T$.

Subcase(iib): Suppose $v_9 \in T$. v_9 strongly dominates v_8, v_{10} and v_{20}. To strongly dominate v_{19} either v_{18} or $v_{28} \in T$.

Subsubcase$(iibi)$: Suppose $v_{18} \in T$. Then v_{18} strongly dominates v_{17}, v_{19} and v_{24}. Thus $|N_s[v_{24}] \cap T| = |\{v_1, v_{18}\}| > 1$. This is a contradiction to G' is strong efficient. Hence $v_{18} \notin T$.

Subsubcase$(iibii)$: Suppose $v_{28} \in T$. Then v_{28} strongly dominates v_{19}, v_{13} and v_6. Now $v_5 \in T$. Thus $|N_s[v_6] \cap T| = |\{v_5, v_{28}\}| > 1$. This is a contradiction to G' is strong efficient. Hence $v_{28} \notin T$. From all the above cases, $v_9 \notin T$.

Subsubcase(iic): To strongly dominate v_5 either v_6 or $v_{27} \in T$.

Subsubcase$(iici)$:Suppose $v_6 \in T$. Then v_6 strongly dominates v_5, v_7 and v_{28}. v_9 strongly dominates v_8, v_{10} and v_{20}.To strongly dominate v_{11}, either v_{12} or $v_{27} \in T$. If $v_{12} \in T$,then

v_{12} strongly dominates v_{11}, v_{13} and v_1. v_{15} strongly dominates v_{14}, v_{26} and v_{16}. v_{18} strongly dominates v_{17}, v_{19} and v_{24}. v_{22} strongly dominates v_{21}, v_{23} and v_{27}. Therefore $v_2 \in T$. This is a contradiction by case(i) $v_2 \notin T$. Hence $v_6 \notin T$.

Subsubcase$(iicii)$: If $v_{27} \in T$, then v_{27} strongly dominates v_5, v_{11} and v_{22}. v_7 strongly dominates v_6, v_8 and v_{26}. If $v_{10} \in T$, then $|N_s[v_{11}] \cap T| = |\{v_{10}, v_{27}\}| > 1$. This is also a contradiction to G' is strong efficient. Hence $v_{27} \notin T$. From the above subcases, $v_3 \notin T$.

Hence there exists no strong efficient dominating set without u. Therefore $\gamma_{se}(G') = 8$ and $S^+(\gamma_{se}(G)) = 1$.

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